Black rings from fluid mechanics

Subhaneil Lahiri

based on `arXiv:0705.3404 [hep-th]` with Shiraz Minwalla
and work in progress with Jyotirmoy Bhattacharya

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Introduction

AdS/CFT maps black holes to deconfined gauge theories (quark-gluon plasmas).

In the long wavelength limit, the gauge theory can by described by fluid dynamics. Some properties of these fluids have been computed by looking at the black holes.

[Policastro, Son, Starinets;...]

We will look at the reverse – using fluid mechanics to study higher dimensional black objects: put them in an asymptotically AdS-ish space.

Black rings

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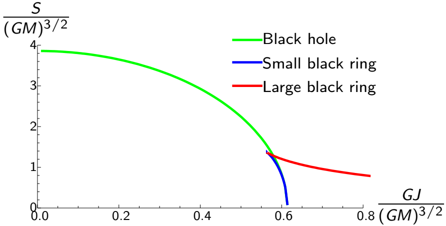
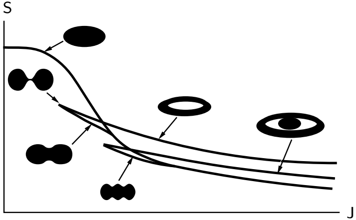
In five dimensions, we are allowed an $S^1 \times S^2$ horizon as well – the black ring. For a range of energies and angular momenta, it is possible to have two black ring and one black hole solutions - violating uniqueness.

[Emparan, Reall]



For $D \geq 6$: no exact solutions (except Myers-Perry). Approximate solutions for $R_{S^1} \gg R_{S^3}$.



[Emparan et al.]

Higher dimensions

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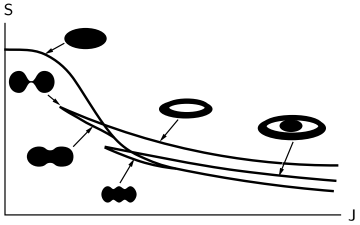

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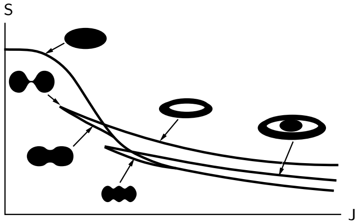


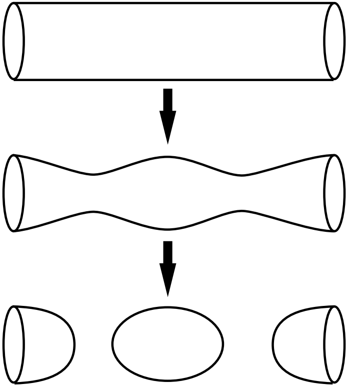
[Emparan et al.]

Other topologies?

[Galloway, Schön]

Gregory-Laflamme instability

Instability of lack strings and branes.



End point of instability?

Related to Plateau-Rayleigh instability? [Caldarelli et al.]

Outline

Plasmaballs in confining theories

Plasmaballs are a generic feature of large N confining field theories that have first-order deconfining phase transitions

Bubble of deconfined phase, surrounded by confined phase, held together by surface tension.

Focus on theories that come from compactifying conformal theories on a Scherk-Schwarz circle.

For concreteness: $\mathcal{N} = 4$ Yang-Mills on $\mathbb{R}^{1,2} \times S^1$ at large N and large λ.
The S^1: $\theta \sim \theta + R_\theta \implies$ confining gauge theory with scale $\Lambda \approx 1/R_\theta$.

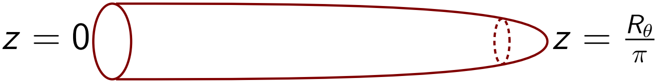
At low temperatures, gravity dual: AdS soliton:

$$\mathrm{d}s^2 = \frac{R_{\mathrm{AdS}}^2}{z^2}\left(-\mathrm{d}t^2 + F_{R_\theta}(z)\,\mathrm{d}\theta^2 + \mathrm{d}\vec{x}^2 + \frac{1}{F_{R_\theta}(z)}\,\mathrm{d}z^2\right),$$

where $F_a(u) = 1 - \left(\frac{\pi z}{a}\right)^4$ and $R_{\mathrm{AdS}}^2 = \sqrt{\lambda}\alpha'$. [Witten]

Small z: Poincaré AdS_5 with one compact direction.

At $z = R_\theta/\pi$, the θ circle contracts: space stops.



Deconfined phase

At high temperatures: the black brane:

$$\mathrm{d}s^2 = \frac{R_{\mathrm{AdS}}^2}{z^2}\left(-F_\beta(z)\,\mathrm{d}t^2 + \mathrm{d}\theta^2 + \mathrm{d}\vec{x}^2 + \frac{1}{F_\beta(z)}\,\mathrm{d}z^2\right).$$

Horizon at $z = \frac{\beta}{\pi}$. Temperature: $\mathcal{T} = 1/\beta$.

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Horizon at $z = \frac{\beta}{\pi}$. Temperature: $\mathcal{T} = 1/\beta$.

Dominant phase above transition temperature, $\mathcal{T}_{\mathrm{c}} = \frac{1}{R_\theta}$.

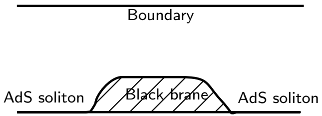
The equation of state of the dual plasma can be found from this gravity solution.

$$\mathcal{P} = \frac{\alpha}{\mathcal{T}_{\mathrm{c}}}\left(\mathcal{T}^4 - \mathcal{T}_{\mathrm{c}}{}^4\right).$$

Plasmaball solutions

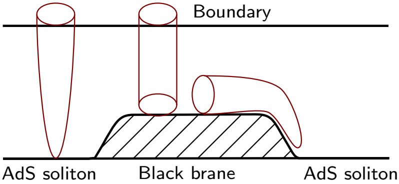
The plasma ball is a bubble of plasma, held together by surface tension, surrounded by the deconfined phase.

On the bulk side, deep interior looks like black brane. Far from the plasmaball, it looks like the AdS soliton. There is a domain wall that interpolates between the two.



In the limit of infinitely large radius, a numerical domain wall solution has been found. The surface tension and thickness can be computed from this solution. [Aharony, Minwalla, Wiseman]

The Scherk-Schwarz circle does not contract in the black brane region but does contract in the AdS soliton region.



Horizon topology: fibre circle over the plasmaball, contracting at surfaces. Disk of plasma $\Longrightarrow$ S^3 horizon.

Fluid mechanics

When the density and velocity of the plasma vary little over the "mean free path" of the quasiparticles (roughly gluons), the effective dynamics of the gauge theory can be described by fluid dynamics.

The equations of motion are $\nabla_\mu T^{\mu\nu} = 0$. The dynamical input is in specifying $T^{\mu\nu}$.

For long wavelengths, we need only go up to one derivative terms.

This approximation breaks down at surfaces – but at scales $\gg$ surface thickness we can replace these regions with a δ-function localised surface tension.

Stress tensor

The zero derivative part

$$T^{\mu\nu}_{\text{perfect}} = \rho u^\mu u^\nu + \mathcal{P}(u^\mu u^\nu + g^{\mu\nu}) \,.$$

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$$\text{where } q^\mu = -\kappa P^{\mu\nu}(\partial_\nu \mathcal{T} + a_\nu \mathcal{T}) \,.$$

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The surface contribution

$$T^{\mu\nu}_{\text{surface}} = -\sigma h^{\mu\nu}\sqrt{\partial f\cdot\partial f}\,\delta(f)\,.$$

surface at $f(x) = 0$.

Three dimensional configurations

We look at rigidly rotating configurations: $(u^t, u^r, u^\phi) = \gamma(1, 0, \Omega)$. The centripetal force is provided by a pressure gradient.

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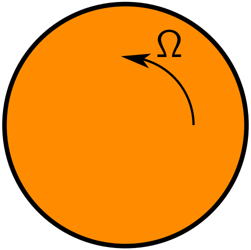
Surfaces: $\mathcal{P} = \pm\frac{\sigma}{r}$. Relates constant of proportionality to Ω and position of surface.

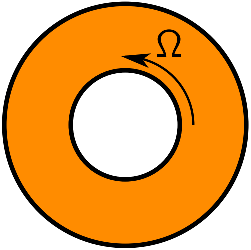
Solutions

We find two types of solution:

Plasmaballs

Plasmarings





B^2

$S^1 \times B^1$

Solutions

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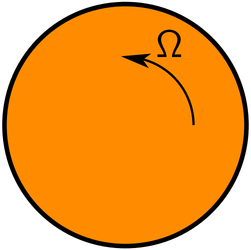

$$\begin{array}{ccc} S^1 & \longrightarrow & S^3 \\ & & \downarrow \\ & & B^2 \end{array}$$

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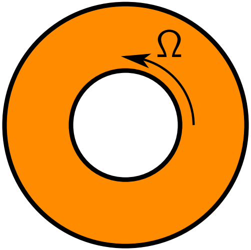

$$\begin{array}{ccc} S^1 & \longrightarrow & S^1 \times S^2 \\ & & \downarrow \\ & & S^1 \times B^1 \end{array}$$

Thermodynamics

We compute the thermodynamic properties of the whole solution with

$$E = \int \mathrm{d}^2 x \left(T^{tt} \right),$$

$$L = \int \mathrm{d}^2 x \left(r^2 T^{t\phi} \right),$$

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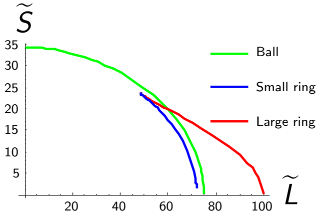
Then we compute an overall temperature and angular velocity via

$$\mathrm{d}E = T\mathrm{d}S + \Omega \mathrm{d}L,$$

we find

$$T = \frac{\mathcal{T}}{\gamma}, \qquad \Omega \text{ as before}.$$

Phase diagram



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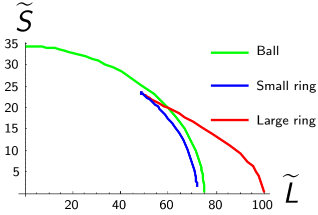
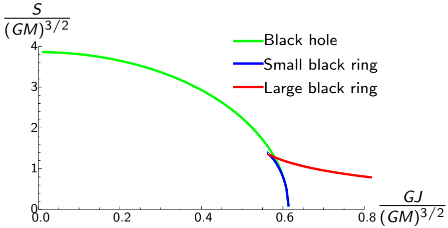




Topologies in six dimensions

S^4

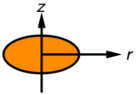
$S^3 \times S^1$

$S^2 \times S^2$

$S^2 \times T^2$

Topologies in six dimensions

$$\begin{array}{c} S^4 \\ \downarrow \\ B^3 \end{array}$$



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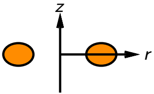

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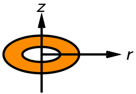
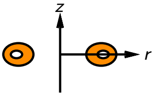
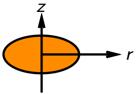


$$\begin{array}{c} S^2 \times T^2 \\ \downarrow \\ B^1 \times T^2 \end{array}$$



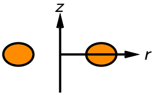
Topologies in six dimensions

$$S^4 \downarrow B^3$$



✓

$$S^3 \times S^1 \downarrow B^2 \times S^1$$



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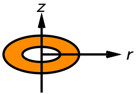
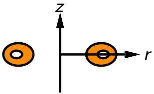


✗

$$S^2 \times T^2 \downarrow B^1 \times T^2$$



✗

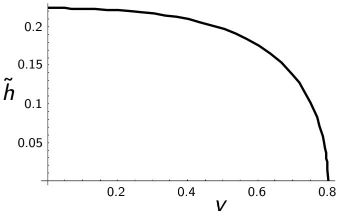
Solving equations of motion

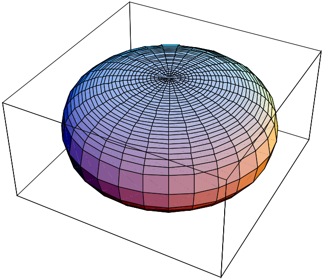
Again: rigid rotation $(u^t, u^r, u^\phi, u^z) = \gamma(1, 0, \Omega, 0)$.

Again: $\frac{\mathcal{T}}{\gamma} = T =$ constant.

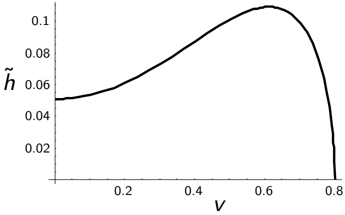
Now: surface satisfies $\mathcal{P} = \sigma K^\mu_\mu$.

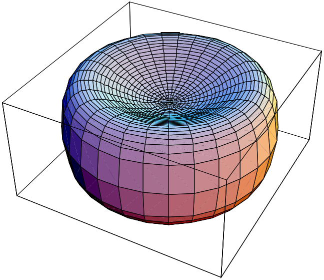
Ordinary balls



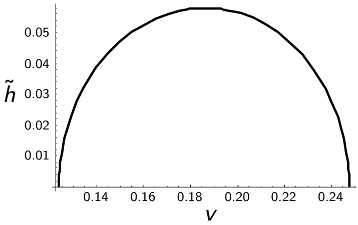


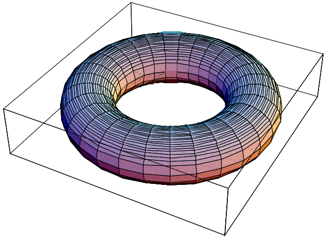
Pinched balls



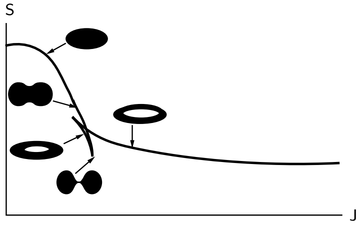


Rings





Phase diagram



[Bhardwaj,Bhattacharya]

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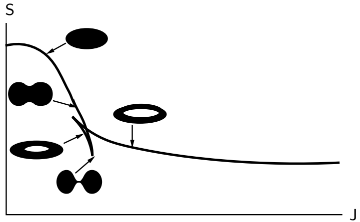

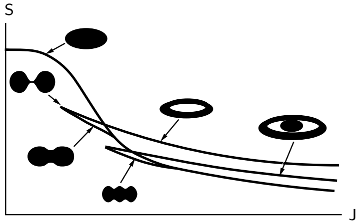

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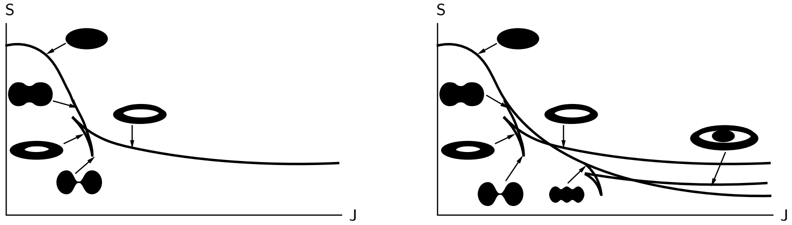


[Bhardwaj,Bhattacharya]

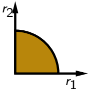
Topologies in seven dimensions

S^5 $\quad$ $S^4 \times S^1$ $\quad$ $S^3 \times T^2$

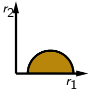
$S^3 \times S^2$ $\quad$ $S^2 \times S^2 \times S^1$ $\quad$ $S^2 \times T^3$

Topologies in seven dimensions

$$\begin{array}{c} S^5 \\ \downarrow \\ B^4 \end{array}$$



$$\begin{array}{c} S^4 \times S^1 \\ \downarrow \\ B^3 \times S^1 \end{array}$$



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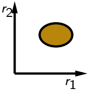
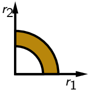
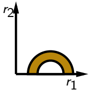


$$\begin{array}{c} S^3 \times S^2 \\ \downarrow \\ B^1 \times S^3 \end{array}$$



$$\begin{array}{c} S^2 \times S^2 \times S^1 \\ \downarrow \\ B^1 \times S^2 \times S^1 \end{array}$$



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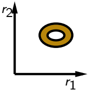


Approximate solutions

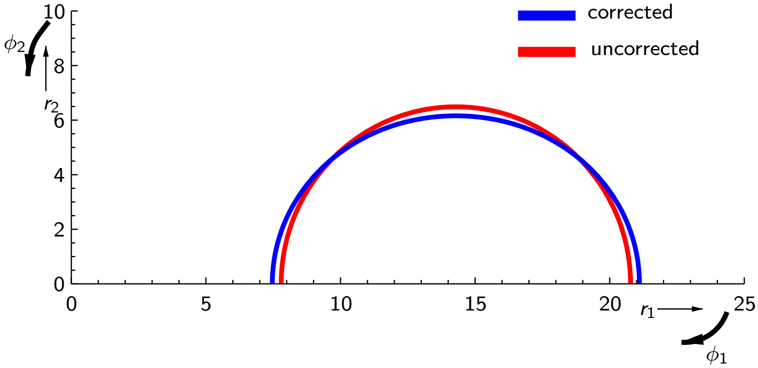
For ring, $B^3 \times S^1$, take $\epsilon = \frac{R_{B^3}}{R_{S^1}}$ small.

For 'torus', $B^2 \times T^2$, take $\epsilon = \frac{R_{B^2}}{R_{T^2}}$ small.

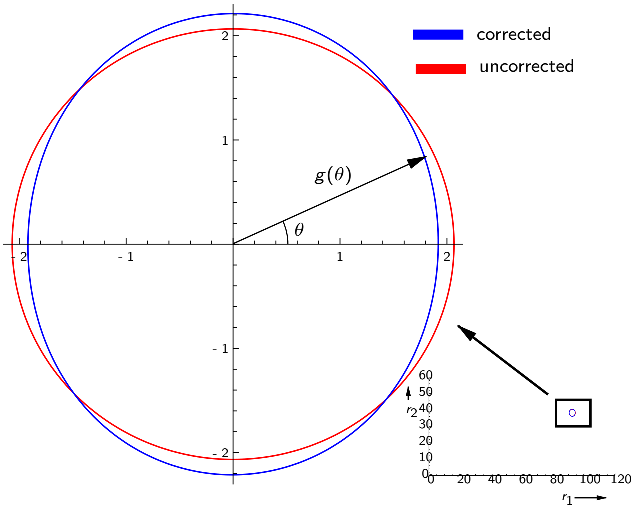
Expand in ϵ. At $\mathcal{O}(\epsilon^0)$ – just a tube.

Similar to black-fold construction of Emparan et al.

Ring



Torus



corrected
uncorrected
2
1
- 1
- 2
- 2
- 1
1
2
$g(\theta)$
θ
60
50
40
30
20
10
0
0
20
40
60
80
100
120
r_2
r_1

Summary

We can get insight to some problems in classical gravity from fluid mechanics in AdS/CFT.

In six dimensions – proposal for phase diagram.

In seven dimensions – new topology.